E 8345 (Pages : 4) Reg. No.....

Name.....

B.Sc. DEGREE (CBCS) EXAMINATION, JANUARY/FEBRUARY 2018

First Semester

Core—FOUNDATION OF MATHEMATICS

(For B.Sc. Mathematics Model I and II/B.Sc. Computer Application Model III–Triple Main)

Time : Three Hours Maximum Marks : 80

Part A

Answer any **ten** *questions.*
Each question carries 2 *marks.*

1. Prove any one of the De Morgan's Laws.
2. What is direct proof ? Use direct proof to prove that square of an even number is an even number.
3. State and verify the absorption laws of logical equivalence.
4. Let $f, g : Z \to Z$ be such that $f(x) = 2x + 3$ $g(x) = 3x + 2$. Find $(f \circ g)(1)$ and $(g \circ f)(1)$.
5. Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find (a) $A \times B$; (b) $B \times A$.
6. Prove or disprove that all real numbers x and y $[x + y] = [x] + [y]$.
7. How many relations are there on a set with elements ?
8. Represent the relation $\{(1, 2), (1, 3), (1, 4)(2, 3)(2, 4), (3, 4)\}$ on $\{1, 2, 3, 4\}$ with a matrix.
9. What are the equivalence classes of 0 and 1 for congruence modulo 4.
10. State the Descartes rule of signs for the positive and negative roots of an equation $f(x) = 0$.
11. Form a rational cubic equation whose roots are $2_1 3 + i$.
12. If α, β, γ are the roots of $2x^3 + x^2 - 2x - 1 = 0$ find the values of $\alpha + \beta + \gamma$, $\alpha\beta + \alpha\gamma + \beta\gamma$ and $\alpha\beta\gamma$.

(10 × 2 = 20)

Turn over

Part B

Answer any **six** *questions.*
Each question carries 5 marks.

13. Find the truth table of $(p \vee (q \wedge r)) \rightarrow (q \wedge r)$.

14. Define :

    (a) Existential Quantifier.

    (b) Universal Quantifier.

    (c) Counter Example.

    (d) Modus Ponens.

    (e) Modus Tollens.

15. Let f be a function from A to B. Let S and T be subsets of B. Show that :

    $$f^{-1}(S \cup T) = f^{-1}(s) \cup f^{-1}(T).$$

16. For any *two* sets A and B, prove that $A - B = A \cap \bar{B}$. Using this result and the set identities, prove that $(A - B) - C = A - (B \cup C)$.

17. Prove that the relation R on a set A is transitive if and only if $R^n \subseteq R$ for $n = 1, 2, 3, \ldots$.

18. Find the ordered pairs in the equivalence relation R produced by the partition $A_1 = \{1,2,3\}$, $A_2 = \{4,5\}$, and $A_3 = \{6\}$ of $S = \{1,2,3,4,5,6\}$.

19. Solve the equation $6x^3 - 11x^2 - 3x + 2 = 0$ whose roots are in H.P.

20. The α, β, γ be the roots of the equation $x^3 - x - 1 = 0$ then find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$. Also find $\Sigma \frac{1+\alpha}{1-\alpha}$.

21. Given equation is $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$. Find the equation whose roots are diminished by -7.

$(6 \times 5 = 30)$

Part C

Answer any **two** *questions.*
Each question carries 15 *marks.*

22. (a) Write the dual of the equation $(a * 1) * (0 + a) = 0$. (3 marks)

(b) Check the validity of the statement using truth table. (6 marks)

If I study, then I will Fail Mathematics

If I do not play cricket, then I will study.

But I failed in Mathematics

Therefore I must have played basket ball.

(c) Define counter example, Prove by using counter example '$\forall x \in R, x^2 \geq x$'. (3 marks)

(d) Check whether $p \vee \neg(p \wedge q)$ a tautology. (3 marks)

23. (a) Let m be a positive integer with $m > 1$. Show that the relation

$R = \{(a, b)\}/a \equiv b \{(\text{mod } m)\}$ is an equivalence relation on the set of integers.

(b) Define partial ordering and total ordering. Show that the divisibility relation on the set of positive integers is a partial order but not a total order.

(c) Suppose the relations R_1 and R_2 on a set A is represented by the matrices $M_1 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

and $M_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. What are the matrices representing $R_1 \cap R_2, R_1 \cup R_2, R_1 \circ R_2, R_2 \circ R_1$

and $R_1 \oplus R_2$.

24. (a) Define a poset and a lattice. Draw the Hasse diagram of the poset $(p(s), \subseteq)$ where $s = \{a, b, c\}$. Is this poset, a lattice, Justify your answer.

(b) Prove that if x is a real number, then $[2x] = [x] + \left[x + \frac{1}{2}\right]$.

25. (a) Prove that every polynomial of degree n has exactly n roots.

(b) Solve $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$ whose root are in A.P.

$(2 \times 15 = 30)$